STATISTICAL ENERGY ANALYSIS (SEA)
Theory and Applications

International Masters Programme in Sound and Vibration
Course: Technical Acoustics II

by

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1 Introduction and historical background

1.1 Introduction

This is a short presentation of the basic theory and procedures for application of a method for modelling dynamical systems called Statistical Energy Analysis (SEA) [1], [2]. The name SEA was established in the early 1960's.

- **Statistical** means that the systems being studied are members of populations of similar design having distributions of their dynamical parameters.
- **Energy** is the primary variable of interest. Dynamic variables such as displacement, pressure, etc., are derived from the energy of vibration.
- **Analysis** is used to say that SEA is a framework of dynamic analysis, rather than a particular technique.

 Statistical approaches in dynamical analysis have a long history. It is emphasised again that the important feature of SEA is the description of the vibrating system as a member of a statistical population or ensemble. The dynamic loads and responses may be random or not.

Traditional analyses of the mechanical vibration of systems such as machines and structures have been focused on the lower few resonant modes. These modes at low frequencies tend to have large displacement response (large strains) and are the candidates for prediction of structural fatigue. Higher order mode vibration of walls, floors, vehicle bodies etc. has been of interest for a long time for sound radiation and propagation. Large, lightweight aerospace and vehicle structures, with high frequency broad-band loads makes higher order modal analysis interesting also for predicting structural fatigue and equipment failure not only noise.

Resonance frequencies and mode shapes of higher order modes show great sensitivity to small variations of geometry, construction and material properties. Modal overlap results in high variability of frequency response functions (FRFs) for such variations [3]. Also, FEM/BEM computer programs used for calculating mode shapes and frequencies are rather inaccurate for the higher order modes. A statistical model of the modal parameters is therefore natural and appropriate when the number of modes in the frequency intervals considered becomes high.

Design engineers have to make dynamic load and response estimates at early stages of development when structural detail is not known or decided. These estimates are made to qualify the design including any isolation, damping, or structural configurations necessary to reduce structural or acoustic response. Highly detailed modelling requires specific knowledge of shape, construction, loading functions, etc. that are not available. Simpler, statistical estimates of response to environment that include main parameter dependence (such as damping, average panel thickness, etc.) are more appropriate at this stage.

**Inspirations for SEA.** Two areas that served as "touch-stones" in early theoretical developments of SEA are the theory of room acoustics, and statistical mechanics.

Room acoustics deals systems of very many degrees of freedom (there may be over a million modes of oscillation of a good-sized room in the audible frequency range) and the interactions between such systems (sound transmission through a wall is an example) using both modal and wave models. The very large number of degrees of freedom is an advantage from a statistical viewpoint - it tends to diminish the fluctuations in prediction of response.
Statistical mechanics deals with the random motion of systems with either a few or very many degrees of freedom. It is random motion of a very special type, which we may call "maximally disordered." In this state of vibration, all modes tend to have equal energy of vibration and to have incoherent motion. The energy of the modes corresponds to system temperature. The state of equal modal energy is spoken of as "equipartition of energy". In SEA, we mostly use the equipartition assumption for modes that resonate in the same frequency band, not for all modes.

Statistical mechanics, and heat transfer, teach us that thermal (random vibration) energy flows from hotter to cooler systems, and that the rate of flow is proportional to temperature (modal energy) difference. This also applies to mechanical dynamical systems excited by broad band noise sources. Narrow band sources are equivalent to broad band sources when system averages are taken. This result can be generalised, with proper care, to pure tone excitation.

Advantages and Limitations of Statistical Analysis. The statistical analysis allows for a much simpler description of the system, by modes or waves. In the former case, modal density, average modal damping, and averages of modal impedance to sources of excitation are required. A wave description uses parameters such as mean free path, surface or volume absorption and general geometric configuration.

The most obvious disadvantage of statistical approaches is that they give statistical answers, with some uncertainty. In very high order systems, this is not a great problem. Many of the systems we apply SEA to, however, may not have enough modes in certain frequency bands to provide predictions with high certainty. We may attempt to calculate the mean response and also calculate the confidence intervals of the prediction [2].

There are also difficulties in the psychology of statistical methods. A designer may predict the structural response of for example car body panels, for which he has engineering drawings, to a loading environment, for which he has operational data. Instead of using a deterministic calculation he may get a "better" estimate if he represents the car body with a SEA model of medium size and reduced complexity, using parameters like panel average thickness and total area!

This simplified statistical model may just as well represent his knowledge of the car body at its 100th or 200th mode of vibration as it is by his drawings. Also, the answers he gets by SEA will be in a form that is usable to him, with parameter dependencies that will allow him to interpret the effect of major design changes on response levels. The uncertainties inherent in his SEA predictions may also reflect the actual variations in response between individual cars that will be present in series production.

1.2 Historical background

In 1959 Lyon calculated the power flow between two lightly coupled, linear resonators excited by independent white noise sources. He found that the power flow was proportional to the difference in uncoupled energies of the resonators and that it always flowed from the resonator of higher to lower resonator energy. About the same time P W Smith found that the response of a resonator excited by a diffuse, broad band sound field reached a limit when the radiation damping of the resonator exceeded its internal damping.

Smith's result was surprising since many workers regarded an acoustic noise field simply as a source of broad band random excitation. When a resonator, excited by a broad band noise, has its internal damping reduced to zero, the response diverges, i.e., goes to infinity. Smith's limit was due to the reaction of the sound field on the resonator, the radiation damping. This limit corresponds to equality
of the resonator energy and the average modal energy of the sound field. If the coupling is strong compared to internal damping, equipartition will result.

Lyon and Maidanik wrote the first paper that may be said to be a SEA publication [4], before the name SEA was coined. Formulas for the interaction of a single mode of one system with many modes of another were developed, and experimental studies of a beam (few-mode system) with a sound field (multi-modal system) were reported. This work showed the importance of the basic SEA parameters for response prediction: modal density, damping, and coupling loss factor.

The earliest application was to sound-structure interaction because it seemed "obvious" that SEA would work best when a sound field, with all of its many degrees of freedom, was involved. Very soon, however, applications were also made to structure-structure interactions.

The earliest work on structure-structure vibration transmission was concerned with electronic package vibration. Aside from ships and aerospace vehicles, the most active uses of SEA have been in building acoustics. A particular area has been the transmission of vibration through structural junctions. SEA was used more and more in the 1970’s to predict structure-borne sound transmission in ships and large buildings. While being out of focus during the 1980’s, SEA is experiencing a revival during the 1990’s with road vehicles, aircraft and noisy machinery among qualified applications.

The calculations of response energies in the SEA subsystems are computationally straightforward, involving the inverse of a well-behaved and robust matrix of coupling loss factors. The number of subsystems may become quite large, with large bookkeeping of information on damping values, coupling loss factors, and mode counts. A number of computer programs have therefore been developed to handle SEA calculations for ships, aerospace structures, and buildings, some of them commercially available and quite advanced.
2 Dynamic analysis of complex structures

One may roughly divide the dynamic analysis of structures after the required complexity of the computational model. See Figure 1. It is obvious that most constructions and structures in our industrialised world are complex and non-homogenous, e.g. buildings, ships, road vehicles, aerospace structures and various appliances and machines. The greater degree of refinement of the product that one strives for, including sound and vibration properties, the more rigorous analysis methods are necessary. The computational methods available for vibro-acoustic analysis and design of complex systems are:

a) Discrete (lumped) mass-spring-damper models
b) Finite-element methods (FEM) or boundary element methods (BEM)
c) Statistical Energy Analysis (SEA)

![Figure 1. Simple classification of vibro-acoustic system models](image)
The structures may also be described and modelled, at least in parts, with experimental data, obtained mainly from

a) Experimental modal analysis (EMA)

b) Frequency response function analysis (e.g. FRF-substructuring)

The finite element method can be illustrated roughly with Figure 2.

**Figure 2.** Dynamic modelling with the finite element method (FEM).

Experimental modal analysis means that the modal parameters (eigenfrequency, eigenfunctions, damping) are derived from measured transfer functions, see Figure 3. Advantages and disadvantages of the FEM method compared to experimental modal analysis are presented in Table 1. This table is also of interest when FEM and SEA are compared.
1. Transfer function measurements with artificial excitation

2. Transfer functions (FRF or IRF)

3. Curve fitting results in modal parameters

4. Animation of modal vectors (mode shapes)

**Figure 3.** Principal work flow for experimental modal analysis (EMA).

**Table 1.** Advantages and disadvantages of finite element analysis

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>The model can be &quot;built&quot; and used before any prototype hardware is available</td>
<td>FEM models can be very difficult and expensive to &quot;build&quot;.</td>
</tr>
<tr>
<td>The model can predict a structure's behaviour under real world dynamic operating conditions</td>
<td>Modelling is generally done by a skilled dynamicist because of the complexities of the available FEM codes.</td>
</tr>
<tr>
<td>An engineer can analytically modify the structure (via the FEM model) much cheaper, faster and easier than he can change actual hardware</td>
<td>Models can be expensive to run, depending on the size of the model. They may also require a large computer for operation</td>
</tr>
<tr>
<td>A model can be, and indeed is often inaccurate</td>
<td>Many implementations cause a user to wait hours before either plotted or printed results are available</td>
</tr>
</tbody>
</table>

Systems have been developed for modelling with combined FEM-calculations and experimental modal analysis (Hybrid modelling). See Figure 4 [5]. This facilitates effective simulation of the impact of modifications to a prototype, provided that the FEM-model can be correlated and updated with the experimental modal model. Updated FE- and hybrid models can of course not be used in very early design phases, such as concept design. Also, only a limited number of vibrational modes can be modelled reliably in this way.
Complex structures are characterised by the following properties:

- Complex geometry for the parts of the structure
- Uncertain dynamic properties and boundary conditions at joints
- Uncertain material properties, many different materials
- Small or moderate damping (very hard to predict)
- Numerous resonances at frequencies some octave above the fundamental resonance frequency

Statistical calculation methods may be as reliable (or unreliable) as detailed deterministic numerical methods (e.g. FEM), see the following chapter. Statistical methods for calculation of the energy distribution will be simpler and smaller, and therefore also cheaper to apply than e.g. FEM when the number of modes that determine the response is large, or at early design stages when details of the structures are undecided. Experimental, detailed modal analysis will also run into difficulties when the modal density and modal overlap becomes higher:

"Modal density describes the frequency proximity of adjacent modes, and thus their susceptibility to precise measurement. At low density an experimental modal analysis is limited by the frequency resolution of the analysis system. At high density, a structure’s dynamics defy modal decomposition by signal processing techniques alone; "zoom" processing proves unable to separate overlapping modal bandwidths and spatial decomposition must be employed. Between these extremes, the analysis is limited by the sophistication and precision of the modal parameter identification algorithms employed."

From "Modal Density - a Limiting Factor in Analysis" by George F Lang, Fox Technology Corporation.

Thin-walled, stiffened structures will often have an “avalanche-like” increase in modal density at a rather small frequency interval, where “local” panel modes start to appear. Figure 5 illustrates this, where one gets so many modes for large ship hulls at around 50 Hz that a dynamic solution of a FE-
model for e.g. a complete aft hull and superstructure will have an unrealistically large extent and produce results with unreasonable and useless detail.

Figure 5. Principal frequency dependence of the cumulated number of vibration modes for a ship hull [3].

It is not quite obvious that one should strive as much as possible for detail in the calculation model. A citation from Maidanik (1977), one of the persons behind the early SEA development is interesting [6]:

"SEA is not alone; experiences in other areas of scientific endeavours have shown that properly chosen sacrifices of details often alleviate difficulties associated with formulating the behavior of models of complex systems. Indeed, excess information can, and usually does, act like noise! This includes not only details that do not admit to practical definitions; it includes even those that can be so defined but are, by and large, irrelevant to the phenomena that are manifested dominantly in the behavior of interest. On the other hand, one must ensure that the loss of details in the descriptions of the behavior of the models would not be too severe to render them insignificant. Indiscriminate sacrifices of details may suppress the very phenomena that contribute to the behavior of the complexes one desires to describe. To prevent such suppressions, and yet to establish models that can be formulated, is a skill that must be acquired to make SEA effective."

Proper approximations rely on knowledge, skill and experience. SEA is a tool for vibro-acoustic analysis of complex systems. The tool works well in the hands of a skilled and experienced analyst. The skill refers mainly to sufficient physical understanding of the structural behaviour and the problem at hand.
3 Variability and predictability of vibro-acoustic systems

Significant variations in vibro-acoustic transfer properties are obtained between individual products that are produced to be identical. Variations of ±5-10 dB for narrow-band transfer functions are usual and typical in serial production of road vehicles, aircraft, ships, appliances etc. at medium and higher frequencies.

Some papers have been published on the variation of transfer function characteristics between e.g. road vehicles [7] and engines [8] with nominally identical design. Kompella et. al. [7] presented measured frequency response functions for a large number of identical vehicles. The FRFs show more random behaviour and scatter as the frequency increases, see Figure 6. By assuming the variability to be due to “insufficient manufacturing quality“, the use of large and highly detailed FE-models may seem to be justified for prediction of dynamic properties, even at frequencies where many modes contribute significantly to the total response.

Figure 6. Magnitudes of the 99 structure-borne FRFs for the RODEOs for the driver microphone [7].

A good summary of the fundamental limiting factors for deterministic modelling and analysis was presented in a recent SEA review paper by Fahy [9]. Apart from the fact that FE- or BE-models become very large at higher frequency due to the finer meshing requirements and that the modelling effort is increasing substantially due to more attention to geometrical detail, the following fundamental limits for prediction of the response in detail exist:

- **Uncertainty about precise dynamic properties.** Sensitivity of eigenfrequencies and phase response to changes in boundary conditions, thickness- and damping distribution etc. increases with mode order.
- **Modal summation.** Contributions from an increasing number of modes are added at each frequency as frequency and/or damping increases.
- **Uncertain dynamic properties of joints.** The dynamic force transmission properties of joints are not very well defined. In addition, dynamic properties of most joints between structural components are especially uncertain at higher frequencies.
• **Uncertain material properties.** Use of alloys, polymers and composite materials makes basic material properties considerably harder to predict for modelling purposes. In addition these material properties will vary much more due to temperature, static loads etc.

• **Uncertain modal damping estimation.** Forced response prediction needs damping to be estimated. For detailed deterministic prediction, either the correct spatial damping distribution or the correct estimates for individual modal damping has to be applied.

One should therefore treat response prediction for multi-modal systems as a probabilistic problem. High-frequency response of a population of nominally similar products of which the individual members differ slightly in many details may be described by an ensemble-average behaviour and a statistical estimation of the distribution of responses around this average.

### 3.1 Theory

The statistics of multi-modal systems was first derived for room acoustics, see e.g., [10]. It has also been studied during the SEA development [11]. Schröder derived some fundamental results already in 1954 [10].

Consider a system where the response at each point and frequency is determined by the sum of a sufficient number of modes with random phase, and where no individual mode is dominating the sum. This will be the case in many dynamical systems above a certain frequency.

Define a logarithmic response function $z$ as [10]

$$z = \ln \frac{x^2}{\bar{x}^2}$$

where $x$ is the response in the system at a certain point and frequency, and $\bar{x}$ is the spatial or frequency average value of $x$.

The standard deviation of $z$, $\sigma = \sqrt{\bar{z}^2 - \bar{z}^2}$ can be calculated if the probability distribution function $W(z)$ is known. The real and imaginary parts of the complex response function are subject to Gaussian distribution

$$W(\text{Re}(x)) = \frac{1}{\sqrt{2\pi \text{Re}(x)^2}} e^{-\text{Re}(x)^2/2\text{Re}(x)^2}$$

(2)

and

$$W(\text{Im}(x)) = \frac{1}{\sqrt{2\pi \text{Im}(x)^2}} e^{-\text{Im}(x)^2/2\text{Im}(x)^2}$$

(3)

where $\text{Re}(x)^2 = \bar{\text{Im}(x)^2}$.

The Gaussian distribution is valid when the response is given as the sum of several complex independent (modal) vectors, of which no one is dominating. This type of response summation is illustrated in Figure 7. Figure 8 illustrates a frequency response function with one dominating component, e.g., a dominating mode or the direct wave field of a strongly damped system. For this case the Gaussian distribution of real and imaginary parts does not apply.
The probability distribution function for $z$, $W(z)$ can now be derived as

$$W(z) = \exp(z - e^i)$$  \hspace{1cm} (4)$$

with $z$ from Equation (1).

The standard deviation $\sigma(z)$ for this distribution is [10]

$$\sigma(z) = \sqrt{z^2 - \bar{z}^2} = 1.28 \text{ (Nepers)}$$  \hspace{1cm} (5)$$

This standard deviation corresponds to $\sigma = 5.57 \text{ dB}$. This is a very interesting, generic result!
3.2 Generic vibro-acoustic models

The theoretical variance given above was derived from a general summation of complex vectors. It should be valid for any dynamic system for which the frequency response function can be expressed as such a sum. For any system with N vibrational modes, the FRF between two points can be expressed as (modal superposition)

\[ H(x, x_e, \omega) = \frac{v(x, \omega)}{F(x_e, \omega)} = \frac{4 j \omega}{M} \sum_{i=1}^{N} \frac{\phi_i(x) \cdot \phi_i(x_e)}{\omega_i^2 (1 + j 2 \zeta_i \omega_i) - \omega^2} \]  

(6)

where \( x \) is the spatial vector, \( x = [x, y, z] \), index \( e \) refers to excitation point, \( M \) is total mass, \( \omega \) is excitation frequency and \( \phi_i \) is the eigenfunction of mode \( i \).

A generic multi-modal dynamic system can be represented by a sum of modes according to Equation (6), where the eigenfrequencies may be distributed for example as

\[ \omega_i = 200 \pi \cdot \log(i+1) \]  

(7)

Eigenfunctions at excitation and response points are random numbers between -1 and 1.

Random shifts in individual eigenfrequencies and modal damping will simulate the influence of material and geometric parameter variations. The eigenfrequencies are shifted as follows:

\[ \omega_{ij} = \omega_{ij0} \cdot (1 + \varepsilon U_{ij}) \]  

(8)

where \( \omega_{ij0} \) is the unshifted eigenfrequency, \( \varepsilon \) is the amplitude of the random variation and \( U \) and \( U_{ij} \) are random numbers with normal distribution (\( m=0, \sigma=1 \)).

Local variations of thickness, mass, boundary conditions etc. results in individual shifts in eigenfrequency for each mode (Equ. 8), where each \( \omega_{ij} \) is shifted by \( \varepsilon U_{ij} \). An ensemble of plates is obtained by using different sets of samples \( U_{ij} \).

The modal damping has the same nominal value, \( \zeta_{ij0} \), for all modes. The uncertainty in damping is modelled by an exponential normal distribution, see equation (9). \( U_{ij} \) has a normal distribution with a mean value of 0 and a standard deviation of 1. An exponential normal distribution is chosen as it provides a realistic damping distribution for the modes.

\[ \xi_{ij} = \xi_{ij0} \cdot 10^{\varepsilon U_{ij}} \]  

(9)

Figure 9 shows the difference in FRF that is obtained for two samples of the generic model.

The modal overlap factor is defined as

\[ MOF = \frac{\pi}{2} n(f) \eta f \]  

(10)

where \( n(f) \) is the average modal density (modes/Hz) and \( \eta \) is the loss factor at frequency \( f \).
Figure 9. FRFs for the generic modal expansion model with $\eta = 5\%$. Standard deviations: 3\% for eigenfrequencies, 30\% for logarithm of the modal damping.

MOF is larger than 1 for $f > 60$ Hz in this example case. Schröder's formulation is applicable when the modal overlap factor is larger than about 2-3. When the modes have approximately equal excitation, no single mode dominates the response in that case and the response is determined by a sum of several modes with different phase and amplitude (interference).

The complex vector contributions from individual modes are shown for 150 Hz in Figure 10 a and b for the two samples. The large differences in phase and amplitude of modal contribution vectors that lead to the 3-4 dB FRF difference in Figure 9 are obvious.

Figure 10. Modal contributions at 150 Hz for the two samples from the generic modal expansion model.

A thin rectangular plate exemplifies a real multi-modal component. The plate has simply supported boundaries and is excited at one point.
For the plate the FRFs are given by

\[
H(x, x', \omega) = \frac{\bar{v}(x, \omega)}{F(x', \omega)} = \frac{4j\omega}{\rho hA} \sum_{i=1}^{\infty} \frac{\phi_i(x) \cdot \phi_i(x')}{\omega_i^2 (1 + 2j\zeta_{ij}) - \omega^2}
\]  
(11)

where

\[
\phi_i(x) = \sin \frac{\pi ix}{l_x} \sin \frac{\pi jy}{l_y}
\]  
(12)

\(l_x, l_y\) are the lengths of the sides of the plate, \(A\) is the area of the plate, \(h\) is the plate thickness, \(\zeta_{ijk}\) is the critical damping ratio for mode \(ijk\).

The eigenfrequency \(\omega_{ij}\) for mode \(ij\) is calculated using the following equation

\[
\omega_{ij} = \sqrt{\frac{E h^2}{12\rho (1 - \nu^2)}} \left[ \left( \frac{\pi i}{l_x} \right)^2 + \left( \frac{\pi j}{l_y} \right)^2 \right]
\]  
(13)

The excitation and response positions on the plate are the same for all plate samples although arbitrarily chosen. This means that the same point-point frequency response function is plotted for all plate samples.

### 3.3 Numeric examples for local parameter uncertainties

Manufacturing processes like rolling, stamping, welding and moulding etc. will introduce localised variations in geometry, thickness, pre-stresses and possibly also material parameters. These local effects will shift individual eigenfrequencies differently depending on how the mode shapes relate to the localised variations of the structure. Other local mechanisms that introduce shifts in individual mode eigenfrequencies are boundary condition variations. These may be due to joint parameter fluctuations as well as variability of connected parts. All these can be represented by random shifts of individual eigenfrequency around the nominal value.

Variations in individual modal damping factors, due to differences in boundary conditions, material and joint damping distribution, sound radiation, etc. can be quite large. Scatter in these factors will cause additional variability of the damping for individual modes. This means that the relations between modal damping factors will also vary between different samples of a plate. This is modelled as randomly distributed damping between the individual modes.

Damping has a significant influence on the FRF, since it will change the amplitudes of the complex mode contribution vectors. Random variation in damping for individual modes will cause largely varying transfer functions.

When the individual eigenfrequencies scatter randomly around their average values, the transfer functions will get very dispersed. This sensitivity to rather small eigenfrequency shifts is explained by the large 180° FRF phase angle jump around the natural frequency of the single-degree-of-freedom system that represents each mode.

We expect a combined scatter of damping and natural frequencies in real structures, and the total impact of these variations on the variance in the FRFs will be a superposition of the respective effects. The necessary variation of the input parameters of the rectangular plate to get randomly varying FRFs
for modal overlap larger than 2-3 is, e.g., a 2% eigenfrequency variation combined with a 20% variation in the logarithm of damping for the studied plate, see Figure 11.

The result obtained, using the combined variations of individual, modal eigenfrequencies and damping shows a good qualitative agreement with the reported measured results for complete cars, compare Figure 11 with Figure 6.

![Figure 11. a) FRFs for combined random variations of eigenfrequencies and modal damping. b) Variation of spatial average velocity level (proportional to the kinetic energy) between different samples of the plate. Standard deviations: 2% for eigenfrequencies and 20% for the logarithm of damping factor.](image)

When energy methods (SEA) are used, spatial average responses for the subsystems are calculated. As shown above, detailed FRF estimation is of a limited practical value, since quite small input parameter uncertainties will lead to low precision in the prediction anyway as soon as the modal overlap is significant. The spatial average energy response is expected to fluctuate much less than the FRFs for corresponding variations in eigenfrequency and modal damping, see Figure 11.b.

There is the same moderate dispersion between samples as for the FRFs at low frequencies. It is meaningful to predict responses from a detailed deterministic model in this frequency range, since it will reveal more detailed information about the structural response than a statistical energy model. However, at frequencies where modal overlap is significant, the energy model predicts the average behaviour equally well.

When a number of simple structural components (subsystems) are connected variations in the matching of the eigenfrequencies of local modes in the different subsystems eventually will make the FRFs between points on different subsystems to scatter more than FRFs between points on the component structures. The energy flow model developed by Fredö [12] was used for two connected plates in a L-configuration as illustrated in Figure 12.

The result of combined variations of individual modal eigenfrequency and damping, corresponding to the result shown in Figure 11a for the simple plate, is shown in Figure 13a. The FRFs between an arbitrary point on plate 1 and a point on plate 2 is shown. The standard deviation in the frequency range with significant modal overlap reaches the same 5-6 dB value as for the single plate. The calculated response energy level in plate 2 with excitation in plate 1, corresponding to the result shown in Figure 11b for the single plate, is given in Figure 13b. The same response energy level has also been calculated with SEA. The SEA calculated result has been included in Figure 13b for comparison. As can be seen, the agreement with the exact analytical result is quite good.
Figure 12. The L-configuration of two simply supported plates used in the analytical model by Fredö [13].

Figure 13. a) Variation of FRF levels between points on two subsystems, calculated for the L-plate. b) Variation of spatial average velocity level in the receiving plate of the L-plate. Combined local parameter variations. Standard deviations: 2% for eigenfrequencies and 20% for logarithm of modal damping.

So, the reliability of deterministic response prediction in road vehicles, aircraft, spacecraft etc. at medium and high frequency is not primarily determined by the size or the geometrical detail of a FE-model or even the modelling skill of the analyst. The limit is set by input parameter accuracy requirements since small variations in eigenfrequency and damping of individual modes will produce large FRF scatter due to overlapping modes. Updating of the FE/BE-model against hardware will not reduce this random error. The needed input parameter accuracy will often exceed reasonable production tolerances, especially when polymer materials and modern assembly techniques are used. Additional cost due to tighter tolerances and the corresponding QA-procedures can only be justified if it results in sufficient additional functionality or customer satisfaction.
4 Overview of general SEA procedures.

The basic theory and equations of SEA can be found in e.g. [2], [13] or [14]. A short review is given here of how SEA calculations are made. It will give a basic understanding about how various concepts of SEA fit together. The SEA prediction procedure can be divided into the four following steps:

1) Modelling of the dynamic system into subsystems and connections (junctions).
2) Determination of SEA-parameters for the model
3) Calculation of energy distribution between subsystems
4) Calculation of average response levels for subsystems

4.1 Model Development

This is the most demanding and important part of the analysis. Unfortunately, computer programs can not yet replace the physical knowledge and experience needed from the analyst in creation of valid SEA models. A good computer program can do most of the work with the other steps. Careful experimental verification is also quite essential when SEA is applied to new product categories, in order to establish typical errors and limits of applicability for the analysis. Nothing really substitutes the personal application experience of a SEA user.

SEA is used to calculate the flow and storage of vibrational energy in a complex, built-up system with both structural and acoustic components (a vibro-acoustic system). The energy storage elements are called subsystems, and should be parts of the system with similar vibrational modes. These modes are usually of the same type (flexural, torsional, acoustical etc.) that exist in some section of the system (an acoustic volume, a beam, a bulkhead etc.) separated by discontinuities from the rest of the structure. Subsystems are often reasonably easy to identify also in complex mechanical or acoustical systems.

Only the resonant vibratory energy of the subsystems is involved in the energy balance calculations. Vibratory energy transmission via non-resonant paths, e.g. the mass-law sound transmission behaviour of a panel below the critical coincidence frequency, must be included as separate coupling elements in the SEA model. The response levels of well-damped subsystems will be under-estimated since non-resonant response is excluded.

Maidanik [16], [17] has suggested a SEA-formulation where non-resonant field contributions are included. It will however increase the complexity of practical modelling unless 3-dimensional geometries can be imported directly from mechanical design (CAD) systems. Also the assumption of conservative coupling between subsystems may be dropped with that formulation.

In selecting the modal groups, we are concerned that they meet the criteria of similarity and significance:

- **Similarity** means that we expect the modes of a group to have nearly equal excitation by the sources, coupling to modes of other subsystems, and damping. If these criteria are met, they will also have nearly equal energy of vibration, "equipartition of energy", and the concept of subsystem modal energy can be used.
- **Significance** means that they play an important role in the transmission, dissipation, or storage of energy. Inclusion of an "insignificant" modal group will not cause errors in the calculations, but may needlessly complicate the analysis. In computerised calculations, such inclusions are of less concern than they might be in hand calculations.
The subsystems are finite linear elastic structures or acoustic cavities, described by their uncoupled natural modes and dissipative losses. Energy is dissipated by system damping, and transferred between the subsystems. By using a number of fundamental assumptions, it is possible to extend the power flow analysis of a pair of oscillators to a pair of oscillator sets (subsystems), see Figure 14.

The assumptions are:
- the oscillators of set 1 are weakly coupled to the oscillators of set 2
- all generalized modal forces must be uncorrelated
- natural frequencies are uniformly probable over a frequency interval $\Delta \omega$
- oscillators in one set have equal energies (equipartition of energy)
- total subsystem energy is the sum of the energies of resonant modes only

Weak coupling does not necessarily mean that the connection between subsystems shall be physically weak. Weak coupling is achieved when the power transmitted out of a subsystem into any connected subsystem is much less than the power dissipated by the transmitting subsystem. High internal loss factors and high wave reflection coefficients at junctions, associated with large wave impedance discontinuities favor this.

The fundamental SEA hypothesis will then be

$$\Pi_{ij} = \Delta N_i \sum_{\alpha, \beta} g_{\alpha \beta} (E_{m,i} - E_{m,j}) = \omega \eta_i \Delta N_i (E_{m,j} - E_{m,j})$$  \hspace{1cm} (14)

where $E_{m,i}$ is the average modal energy (= energy/mode) of subsystem $i$

$g_{\alpha \beta}$ is the oscillator-oscillator coupling coefficient

The energy flow between two subsystems may also be expressed as

$$\Pi_{12} = \omega \cdot \eta_1 (f) \cdot \Delta f \cdot \eta_{12} \cdot \left[ E_{m1} - E_{m2} \right]$$  \hspace{1cm} (14a)

where $E_{m1}, E_{m2}$ are the modal energies (energy/mode) of the subsystems

$n_1$ is modal density for subsystem 1

$\eta_{12}$ is the coupling loss factor
This shows that the energy will always flow from subsystems with the higher average modal energy to subsystems with lower modal energy. This is the basic coupling equation in the classical SEA calculations.

The following "reciprocity" relation for the coupling loss factors follows from applying the basic SEA assumptions

\[ n_i \eta_{ij} = n_j \eta_{ji} \]  

(15)

Excitation by noise and vibration sources will supply vibratory power to some of the subsystems. The energy from these sources is either dissipated by mechanical damping in the subsystems, or transferred between them (coupling losses).

The coupling, excitation and dissipation for two subsystems may be illustrated in a "network" diagram shown as in Figure 15.

![Block diagram for a two-subsystem SEA model.](image)

**Figure 15.** Block diagram for a two-subsystem SEA model.

What do these subsystems represent? Obviously they represent an entire ensemble of structures with the same main geometry parameters (volume, area, thickness etc) and material parameters, but undefined details about shape, boundary conditions etc. This can be illustrated by Figure 16, showing an ensemble of plates that will be modelled as a subsystem with exactly the same parameters!
4.2 Establishing the SEA Parameters

The SEA parameters to be determined for the subsystems and junctions are the following:

1) Input powers $W_i$ to the $i$-th subsystem
2) Modal densities $n_i$ for the $i$-th subsystem
3) Internal loss factors $\eta_i$ for the $i$-th subsystem.
4) Coupling loss factors $\eta_{ij}$ between the $i$-th and the $j$-th subsystems

**Input power definition.** The input power to subsystems may either be specified directly or derived from a dynamic load quantity when subsystem parameters are known. It may be the input power from a point force acting on a plate or from an acoustic reverberant field acting on a plate. More details are presented in the examples later on.

**Modal density.** The energies determined by SEA are averaged over a frequency band. The number of modes in that band will determine the variance to be obtained in the resulting frequency averaged energy. The number of resonant modes can be estimated from the modal density, defined as

$$n_i(f) = \frac{\Delta N}{\Delta f} \text{ (modes/Hz)}$$

Where $\Delta N$ is the number of modes in the frequency band $\Delta f$.

The modal density is mostly determined theoretically, and tables of asymptotic formulas for basic structures are given for example in [2] or [14].

The modal density of an acoustic cavity (or another 3-dim subsystem) is given as

$$n(f) = \frac{\omega^2 V}{\pi c_p^2 c_g} + \frac{\omega A}{4\pi c_p c_g} + \frac{P}{8 c_g} \text{ (modes/Hz)}$$

where $V$ is the volume, $A$ is the surrounding area, $P$ is the total perimeter of all surrounding surfaces, $c_p$ is the phase speed and $c_g$ is the group speed.
The modal density of a plate (or another 2-dim subsystem) is, with the same notations,

\[ n(f) = \frac{A \cdot \omega}{c_p \cdot c_s} + \frac{P}{c_s} \]

(18)

and the modal density for the subsystems of a beam or rod (or another 1-dim subsystem) is

\[ n(f) = \frac{2 \cdot L}{c_s} \]

(19)

The modal densities (average number of modes/Hz or modes/rad) for subsystems, are usually calculated by the SEA computer programs directly after input or update of subsystem data.

**Dissipative loss factors.** The internal, dissipative energy losses due to damping etc. for each subsystem is described by the loss factor, defined as

\[ \eta_i = \frac{\Pi_{i,\text{diss}}}{\omega E_i} \]

(20)

where \( \Pi_{i,\text{diss}} \) is the dissipated power in subsystem \( i \)

\( E_i \) is the stored vibrational energy in subsystem \( i \)

They represent the relative amount of dissipative losses compared to the stored elastic energy in each subsystem. The data has to be supplied for each subsystem as empirical data, or as predicted loss factors of certain damped structures [15].

**Coupling loss factors.** The power flow into connected subsystems can be defined analogously to Equ. (17) by a coupling loss factor (CLF) \( \eta_{ij} \). Formally this means that

\[ \eta_{ij} = \frac{\Pi_{ij}}{\omega E_i} \]

(21)

where \( \Pi_{ij} \) is the power flowing from subsystem \( i \) to subsystem \( j \)

\( E_i \) is the stored total energy in subsystem \( i \) when \( E_j = 0 \).

Coupling loss factors are related to other quantities of junctions [2]. The CLF:s are easily derived from the *sound transmission loss (STL)* for partitions that separate acoustic cavities. For acoustic-structural coupling, CLF:s may be derived from the *radiation efficiency or radiation resistance*. CLF:s for junctions between plate-like subsystems relate to *wave transmission coefficients* for the junction. For structural junctions that define a point connection, the CLF may be derived from the *mechanical mobilities* of the connected subsystems.

The coupling loss factors (CLF:s) for different energy transmission paths can be calculated for a number of junction types from published formulae [2], [13], [14].
4.3 Calculation of the energy distribution.

A set of simultaneous energy balance equations is used to calculate the resulting stationary energy levels of each subsystem. Response levels in engineering quantities (pressure, stress, acceleration, etc.) can then be easily calculated from these energies, see the next section.

When the SEA-parameters for all subsystems and junctions have been determined, the set of energy balance equations will be completely defined. This may be written as the following matrix equation

\[
\Delta f \cdot \omega \left[ \begin{array}{cccc}
   n_1 \eta_{1_{\text{tot}}} & -n_1 \eta_{1_{\text{tot}}} & \cdots & -n_1 \eta_{1_{\text{N}}} \\
   -n_2 \eta_{2_{\text{tot}}} & n_2 \eta_{2_{\text{tot}}} & \cdots & -n_2 \eta_{2_{\text{N}}} \\
   \cdots & \cdots & \ddots & \cdots \\
   -n_N \eta_{N_{\text{tot}}} & -n_N \eta_{N_{\text{tot}}} & \cdots & n_N \eta_{N_{\text{tot}}} \\
\end{array} \right] \left[ \begin{array}{c}
   E_{m_1} \\
   E_{m_2} \\
   \vdots \\
   E_{m_N} \\
\end{array} \right] = \left[ \begin{array}{c}
   W_{m_1} \\
   W_{m_2} \\
   \vdots \\
   W_{m_N} \\
\end{array} \right]
\]

or in matrix notation

\[
\Delta f \cdot \omega [A] \cdot \{E_m\} = \{W_m\} \quad (22)
\]

The matrix \([A]\) is real, symmetric and positive definite of size \(N \times N\) where \(N\) is the number of subsystems. By appropriate numbering of subsystems, the matrix may be kept significantly banded, which improves the calculation speed and extends the limit of maximum number of subsystems in the calculations.

4.4 Calculation of average response levels.

Total energies in each frequency band are obtained by multiplying the calculated modal energy levels with the calculated number of modes in the band. From these total energies, average response quantities are determined. For example, the relationship between the total energy of an acoustic subsystem (cavity) and the spatial average sound pressure is

\[
E = \frac{\langle p^2 \rangle}{\rho c^2} V
\]

and the relationship between total energy in a structural subsystem and the spatial average vibration velocity is

\[
E = M \langle v^2 \rangle
\]

where \(M\) is the total mass of the substructure.
5 SEA application examples

Statistical energy analysis has been used for a long time to predict noise and vibration problems in aerospace-, shipbuilding- and building applications. Lyon, Eichler, Smith and Scharton [18]-[21] investigated structure-borne sound transmission and coupling between acoustic and bending wave fields in the 1960’s in a series of studies. The method was used early to predict acoustically induced vibrations of electronic equipment in satellites and warheads [22], [23]. See also Figure 17. Also calculation of sound insulation of electronic boxes for sensitive equipment was developed during the 1960’s [24].

![Figure 17. Example of early SEA applications in aerospace.](image)

In building acoustics, Kihlman [25] derived the transmission coefficients for a junction between four plates, taking all wavetypes into account, which was used to estimate the power flow across such junctions. The method used is closely related to SEA, although SEA terminology was not used explicitly. The use of SEA for prediction of sound transmission loss of single and double walls was presented early in a series of publications by Crocker et.al. [26], [27], [28].

Several early and fundamental studies were carried out on vibration propagation prediction in simple coupled structures using SEA during the 1960’s, which became a foundation for larger projects involving prediction of internal vibration levels of aircraft [29],[30] and satellite [31], [32] electronics. Figure 18 shows a typical SEA model for such a structure, the number of subsystems was still limited to about 10-15.

![Comparison of estimated and measured acoustic acceptance function $A_b$ for rocket. The internal loss factor $\eta_m$ is assumed equal to $10^{-3}$.](image)

The first publishes ship application of SEA was 1969 [33]. SEA formalism was used to diagnose structure-borne transmission paths from modal energy relations between subsystems. This is illustrated in Figure 19. The flow of energy is from subsystems with higher modal energy to systems with lower.
The use of large SEA models (several hundred subsystems) was developed and tested during the 1970’s for prediction of structure-borne transmission in ships and large buildings, see e.g.

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**Figure 18.** Example of an early SEA model for satellite electronics.

**Figure 19.** First ship application of SEA. Transmission path diagnosis from modal energies.
Examples of models are shown in Figures 20 and 21. The complexity of these models could vary, depending on which wave types that are included.

These SEA models have proved to be quite useful, with acceptable engineering accuracy for frequencies over approximately 100 Hz for ships and approximately 200 Hz for concrete buildings. These studies also indicate that the actual number of resonant modes in each subsystem does not have to be large when a large number of subsystems are involved.

A renewed and stronger interest to use SEA for noise and vibration prediction has appeared during the later years of the 1980’s. The very strong optimism created during the 1970’s about FEM as a universal tool for computation of dynamic response also in vibro-acoustic systems with a very large number of modes has been replaced by a more realistic attitude. More people realise that the limitations of reasonable use of FEM/BEM are not primarily set by computer resources, see e.g. chapter 3 of these notes. It is not hard to consider FEM and SEA as complementary tools, where FEM is outstanding for analysis of “global”, low order modes and SEA can be used for estimations of “local” vibration energy of subsystems.
Figure 21.

SEA was used early for calculation of sound insulation of walls. Successful calculations of sound transmission loss of more complex wall designs, e.g. stiffened panels and triple walls [39] have been performed in the 1980’s, see Figure 22.

Figure 22.

Fig. 4. Measured and calculated transmission loss of a cross-stiffened 4 mm aluminium panel.\(^7\) • • • Measured; × • × Calculated from eqns (3), (8) and Fig. 2; • • • \(R_u\) (eqn. (8)); ▲ ▲ ▲ \(R\) (eqn. (3)).
SEA-predictions have also been applied in European development programs for satellites and space electronics during the 1970’s and 80’s, see Figure 23 [40], [41]. New methods for measurement of coupling loss factors and modal densities have been developed and used.

\[ \text{Figure 23} \]

SEA modelling is used in a very interesting way in [42] to calculate the resulting dissipation loss factor of sand-filled structures. The sand is modelled as an acoustic medium with losses and fluid-structure coupling is used for the sand volume and the structure. The same method is useful for the estimation of loss factors for light panels in the vicinity of porous absorbers. The friction between the sand particles and the structure can be neglected without introducing serious errors.

The first published paper on SEA applied to a car appeared in 1985 [43]. The model used was strongly simplified, see Figure 24. The results obtained were encouraging in spite of that, see the examples in figure 25.
The entire automotive industry has shown a rapidly growing interest in SEA during the 1990’s, see e.g. [44], [45], [46] and [47]. It is especially interesting to use SEA as an optimisation tool especially since demands on light-weight design increase continously. Experimental (test-based) SEA [48], [49] has been developed during the late 1980’s and the 1990’s as a complement to analytical (predictive) SEA especially to be applied to automobile development. The use of experimental SEA is not without complication [50] and restricted essentially by the same assumptions as the “analytical” SEA. It therfore needs further evaluation and development.
Figure 7. SEA ‘Thermogram’ 1 kHz Front Shocktower Force
6 References


